QUBE-Servo 2 – Position Control

1 Background

1.1 Motivation

One of the most common tasks that automation, robotics, and industrial engineers are called upon to perform when creating industrial systems is to control the position of a DC motor. From automation in manufacturing, to automotive systems, autonomous systems and even aerospace, DC motors are used to actuate critical systems, and their position needs to be controlled to perform within specific design criteria. As an introduction to control systems, the control of a DC motor serves as an excellent starting point because DC motors are relatively easy to model and control.

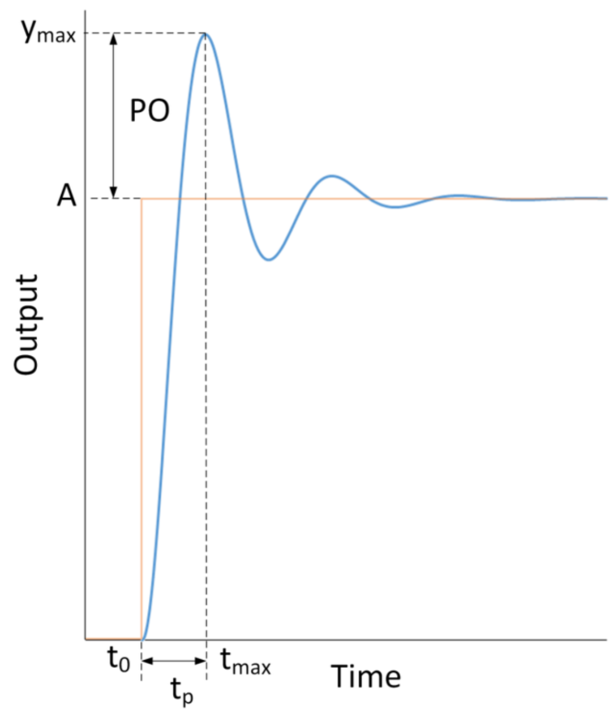
In this lab you will design and tune a proportional-derivative (PD) controller to maintain a desired angular position of the inertial disc. For a simple system such as a DC motor, a common approach to designing a position feedback controller is to start with a theoretical model of the system. As a first step you derive a mathematical model, e.g. a transfer function, based on the theoretical gain and time constant of the system. These theoretical system parameters are then used to calculate desired proportional and derivative gains to achieve a certain system response (e.g. overshoot and risetime). In practice, this quantitative approach to determining controller gains only provides you with a starting point. Often, given the unmodeled components in the mathematical model (e.g. Coulomb friction), you will need to manually tune your controller gains in order to achieve an actual system response that meets your requirements.

1.2 Key Concepts

Before we conduct the experiment, let us review a few key concepts. Recall that the standard second-order transfer function has the following form:

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{1}$$

where ω_n is the natural frequency and ζ is the damping ratio. The behavior of its response depends on the values of its natural frequency and damping ratio. The typical response when applying a step input of amplitude A to a second-order system is shown in the figure below.



Response of a second-order system to a step input

The maximum value of the response is denoted by the variable y_{max} and it occurs at a time t_{max}. The percentage of overshoot (PO) quantifies the maximum amount of overshoot the system exhibits, and can be calculated as follows:

$$PO = 100\left(\frac{y_{max} - A}{A}\right) \tag{2}$$

The peak time of the system, t_p, is defined as the time it takes for the response to reach its maximum value, t_{max}, from the initial step time, t_0:

$$t_p = t_{max} - t_0 \tag{3}$$

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the following equation:

$$PO = 100e^{\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)} \tag{4}$$

The peak time, however, depends on both the damping ratio and natural frequency of the system and it can be derived as:

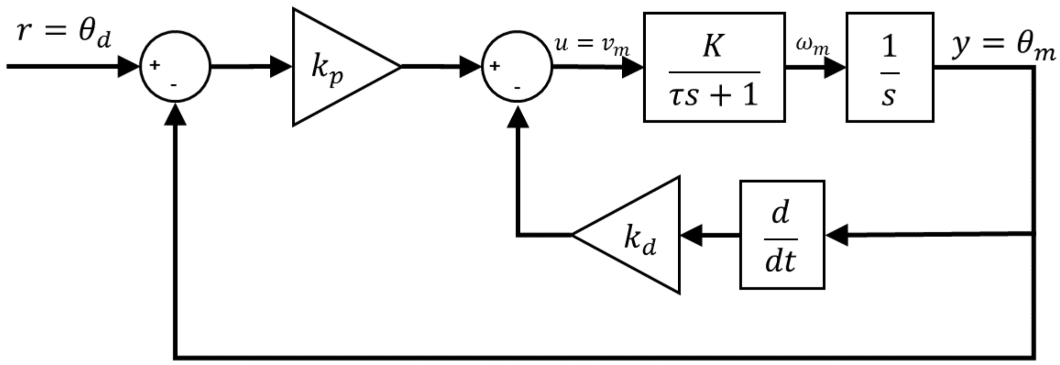
$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{5}$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

The QUBE-Servo 2 voltage-to-position has the following transfer function:

$$P(s) = \frac{\Theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)} \tag{6}$$

where K is the steady-state gain and τ the time constant of the system. In this lab we will use a slight variation of the classic PD control, called rate feedback control as illustrated below.



Block diagram of rate feedback control

Unlike the standard PD, only the negative velocity is fed back (i.e. not the velocity of the error). The rate feedback position controller for the QUBE-Servo 2 has the following transfer function:

$$\frac{\Theta_d(s)}{\Theta_m(s)} = \frac{Kk_p}{\tau s^2 + (1 + Kk_d)s + Kk_p} \tag{7}$$

where the proportional and derivate controller gains are k_p and k_d respectively, K is the steady-state gain, and τ the time constant of the system.

By rearranging Equation 7 to match the prototype second-order transfer function shown in Equation 1, k_p and k_d can be expressed as follows:

$$k_p = \frac{\omega_n^2 \tau}{K} \tag{8}$$

$$k_d = \frac{2\zeta\omega_n\tau - 1}{K} \tag{9}$$

2 LAB PROCEDURE

Let us assume that in an earlier activity you have mathematically derived the voltage-to-position transfer function of the DC motor of the QUBE-Servo 2 based on its equations of motion as follows:

$$P(s) = \frac{\Theta_m(s)}{V_m(s)} = \frac{23.8}{s(0.1s+1)} \quad (10)$$

The system has a steady-state gain of K = 23.8 rad/s/V and a time constant of τ = 0.1 seconds. Let us further assume that you are required to design a PD position controller that has an overshoot of less than 5% and a peak time of no more than 0.2 seconds.

1. Using Equations 4 and 5 determine the required natural frequency (ω_n) and damping ratio (ζ) that will satisfy the overshoot and rise time requirements of the controller.
    a. What does the natural frequency of the system quantify?
        i. It is the frequency at which the system tends of oscillate when continuously subjected to an external harmonic force
        ii. It quantifies the frequency at which the system tends to oscillate in the absence of any driving force
        iii. None of the above
    b. Based on the damping ratio that you have calculated, what category best describes the system?
        i. Underdamped system
        ii. Critically damped system
        iii. Overdamped system
2. Using Equations 8 and 9, as well as the system parameters given above, calculate the corresponding theoretical control gains k_p and k_d that will satisfy your controller requirements.
    a. What gains did you find? Select the answer that is closest to your calculations.
        i. $k_p = 0.98$ and $k_d = 0.08$
        ii. $k_p = 1.98$ and $k_d = 0.08$
        iii. $k_p = 2.98$ and $k_d = 1.32$
3. Use the simulation below to verify that your calculated gains result in the required overshoot and peak time using your modeled system as given in Equation 10. Set the toggle switch to **Modelled** and use the **gain sliders** to enter your calculated gain values. Press the **Start** button and examine the system's response to a square wave with an amplitude of +/- 0.5 rad and a frequency of 0.4 Hz.
4. Once you have determined that your theoretical gains result in a satisfactory response, let's apply the same gains to a simulated version of an actual QUBE-Servo 2. While the simulation is running, set the toggle switch to **Actual** and examine the system's response.
    a. When using the theoretical gains, did you notice any differences between the modeled response and the actual response? Select all that apply.
        i. Actual response has steady-state error.
        ii. Actual response has a different peak time and overshoot.
        iii. Actual response was similar to modeled response.

5. Measure the percent overshoot and peak time of the your QUBE-Servo 2 response. If they do not meet the desired specifications, use the gain sliders to manually tune your controller gains in order to get as close as possible to the design requirements.

3 ASSESSMENT

1) Show details of how you calculated the required system natural frequency and damping ratio for your PD controller.
By rearranging Equation 4 and assuming PO = 5, first determine the required damping ratio as follows:

$$\zeta = \frac{-\ln\left(\frac{PO}{100}\right)}{\sqrt{\ln\left(\frac{PO}{100}\right)^2 + \pi^2}} = 0.69$$

The required natural frequency can then be determined by substituting t_p =0.2 and ζ into Equation 5:

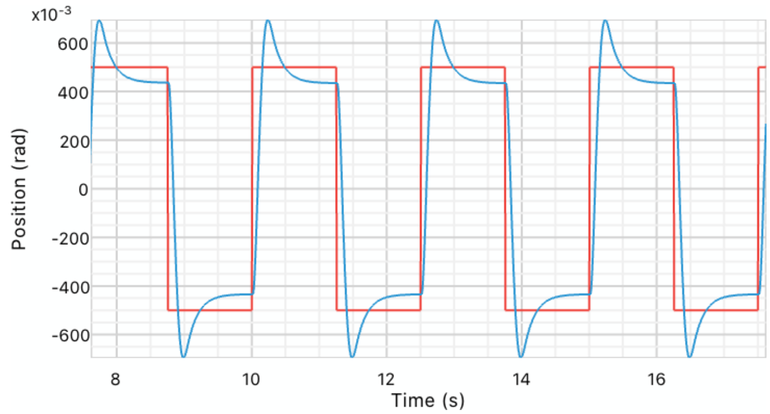
$$\omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}} = 21.7\ rad/s$$

2) Show detailed calculations of how you derived the theoretical PD gains for your controller.
Assuming K = 23.8 and using the parameters calculated previously in Question 1, calculate the required PD gains using Equations 8 and 9 as follows:

$$k_p = \frac{\omega_n^2 \tau}{K} = 1.98$$

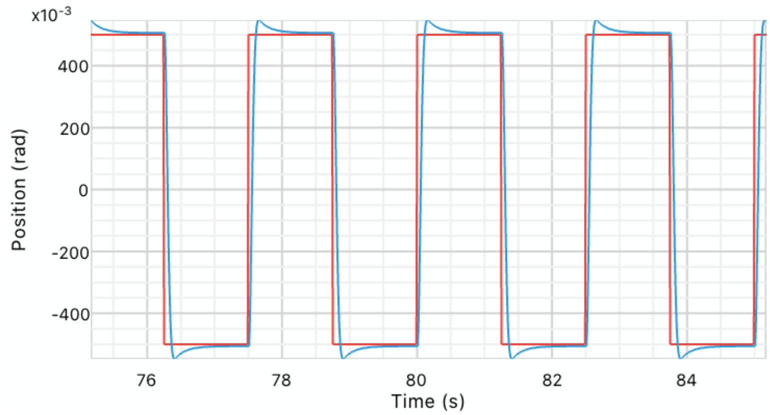
$$k_d = \frac{2\zeta\omega_n\tau - 1}{K} = 0.084$$

3) What effect do the proportional and derivative gains have on the response of the system?
A larger proportional gain increases the response of the system and as such reduces the peak time, but also causes a higher overshoot. A larger derivative gain reduces overshoot.
4) Using the theoretical gains, what percent overshoot and peak time values did you measure from the actual system response? What were the tuned gains?
The screenshot below shows sample results of the actual response of the system using the theoretical gains as calculated in Question 2.



Results of the actual response using theoretical gains

These gains result in an overshoot of approximately 20% and a peak time of 0.2 seconds, and thus need to be tunes in order to meet the required controller criteria. By manually tuning both the proportional and derivative gains, e.g. k_p = 4.32 and k_d = 0.209, we can achieve an overshoot of 4% and a peak time of 0.2 seconds as shown in the screen shot below.



Results of the actual response using tuned gains

5) If observed, comment on why there is steady-state error in the actual response of the simulated QUBE-Servo 2, while the modeled response shows none. How would you eliminate the steady-state error?

The results using the tuned controller gains show negligible steady-state error. However, if present, larger steady-state errors can be minimized by using an integral term, i.e. implementing a full PID controller.